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# SOLITONS IN THE IONOSPHERE – ADVANTAGES AND PERSPECTIVES

MIROSLAVA VUKCEVIC and LUKA Č. POPOVIĆ

*Astronomical Observatory, Volgina 7, 11060 Belgrade, Serbia*
E-mail: mvukcevic@aob.rs, lpopovic@aob.rs

**Abstract.** We present the recent work on the soliton formation possibility within the ionospheric layers of the Earth. Linear waves are investigated very well and their existence is confirmed in the huge literature. However, they are unable to explain some of the fine structures observed in the Earth's atmosphere and ionosphere. Detection of the ion/electron density drop and consequently the density drop of the neutral gas is difficult to explain by linear theory. Therefore, we employ nonlinear magneto-hydro-dynamic (MHD) theory, investigating the perturbation of compressible fluid under the influence of self-gravity, rotation and magnetic field in the plain geometry. Solution that we search for is soliton, stable wave of constant amplitude and group velocity. Such a solution is more accurate and its space and time localization give an opportunity for instant detection. We have derived necessary condition for the vortex type of the solution as a balance of dispersive and nonlinear effects. At higher latitudes dispersion is mainly driven by rotation while near the Equator magnetic field modifies the solution within the E and F layer. This very general description of the ionosphere provides the conclusion that the unperturbed layer thickness cannot be taken as an ad hoc assumption, it is rather a consequence of the equilibrium property.

## 1. INTRODUCTION

Solitons are solution of nonlinear equations, in general. There are one dimensional solitons (ocean surface gravity waves, spiral density waves) and two-dimensional solitons (vortex, hurricane). They are stationary solutions as a balance between nonlinearity and dispersion. It can be concluded from the linear dispersion relation on the dispersive properties of the system and it can give a hint for the type of integrable nonlinear equation. Dispersive properties are consequence of the frequency dependence on the wave number, meaning that different wave numbers are traveling with different velocities. It results in the fact that the top of the wave travels faster that the bottom, braking the wave as it is shown in Fig. 1.

**Figure 1:** Dispersive one-dimensional ocean surface wave.

Under certain circumstances, this dispersion is balanced by the nonlinear effects and that circumstances are defined by equilibrium values. If the soliton is possible to be created than these equilibrium parameters could be used to derive the size and amplitude of the structure.

In the case of ionosphere we derive the nonlinear equation that has two type of nonlinearity: vector and scalar. Vector term is responsible for some turbulent dynamics known as Hasegawa-Mima equation and Rossby waves. Scalar term will be responsible for the solitary vortex creation (Vukcevic & Popovic 2020; Vukcevic, 2019).

## 2. IONOSPHERE PROPERTIES

Ionosphere is divided in three layers, in general. D layer, located at 50-100 km from the Earth's surface, charged particles contribution can be neglected so that ponderomotive force effects are small compared to Coriolis force effects (Gershman, 1974). E layer is located at 100-150 km and ponderomotive force is on the order of the Coriolis one. F layer is at heights of 150-400 km with the dominant ponderomotive force.

We have used the following plasma conditions within the ionosphere: ions are unmagnetized so the ion velocity is the velocity of the neutral gas; ion velocity across the magnetic field is equivalent to gas velocity; ions are dragged by neutral gas completely while electrons are magnetized and frozen in external magnetic field; electron velocity is defined by ExB (Kaladze et al., 2004); viscous effects are neglected due to high Hartmann number (Kaladze et al., 2004).

Local coordinate system of the ionosphere layer is defined as in Fig. 2.

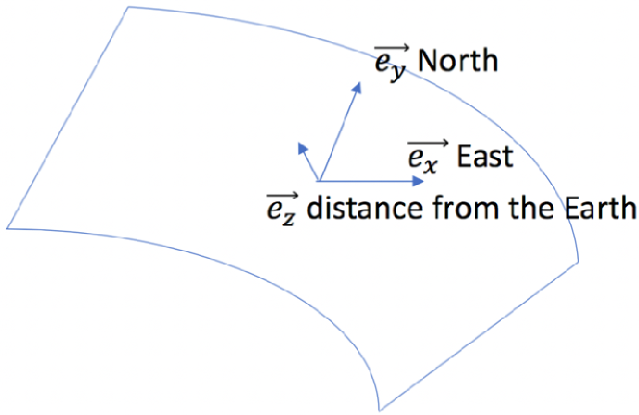


**Figure 2:** Local coordinate system of the ionosphere layer. In the horizontal plane, there are two axes x and y. Along the z axis is defined the distance of the horizontal plane from the Earth's surface.

Earth's angular velocity has the following components: $\Omega=\Omega(0,\sqrt{1-b^2},b\Omega)$, where the Equator is defined by b=0, while the pole is defined by b=1. Geomagnetic field is assumed to be magnetic dipole with components: $B_0=B_0(0,\sqrt{1-b^2},-2bB_0)$.

## 3. NONLINEAR EQUATION AND SOLUTION

Using set of standard fluid equations (continuity equation, momentum equation accompanied with Poisson's equation), non-constant thickness of the layer and drift approximation, we have derived following nonlinear equation

$$-0.2uf\frac{\partial}{\partial y}(B\phi - A\nabla^2\phi) - \frac{1}{2}B'\frac{\partial}{\partial y}\phi^2 + A(\nabla\phi\times\nabla)_z \quad \nabla^2\phi + +(\phi_0' B - \sigma_0')\frac{\partial}{\partial y}\phi - \phi_0' A\frac{\partial}{\partial y}\nabla^2\phi = 0, \tag{1}$$

with relevant frequencies

$$f = f_R + f_H = 2b\left(\Omega + \frac{enB_0}{\rho}\right)\left[v_x, v_y, 0)\right] \quad - \frac{2enbB_0}{\rho}\left[5(1-b^2)v_x, (1-2b^2)v_y, 0)\right] . \tag{2}$$

Normalization of the variables are done by $2\Omega+H$ where $H=enB_0/\rho$. Here n is number of ionized particles while ρ is density of neutral gas.

Solution of the nonlinear equation has a form

$$\phi = \frac{2\lambda}{\nu}F(R), \tag{3}$$

where R=$\sqrt{\lambda}r$ is dimensionless radius of the structure in the moving frame, and F is the solution of following equation (Zakharov, Kuznetsov; 1974)

$$F = 2.4\left(\cosh\left(\frac{3}{4}R\right)\right)^{-\frac{4}{3}}. \tag{4}$$

Parameters

$$\lambda(x) = \frac{1}{A}\left(B - \frac{\sigma_0'}{u}\right), \quad \nu(x) = \frac{1}{2u}\left(\lambda A' + \frac{\sigma_0''}{u}\right), \tag{5}$$

are defined by gradient and second derivative of density with respect to x.
Potential has a form of solitary vortex traveling along y coordinate (northward) with constant velocity u.
If the normalization factor is not symmetric, solution is

$$F = \cosh\left(\frac{3}{4}R(1 + f(x)\Lambda)\right)^{-\frac{4}{3}}. \tag{6}$$

Soliton vortex is asymmetric elongated either along x or along y axis, depending on the frequency.

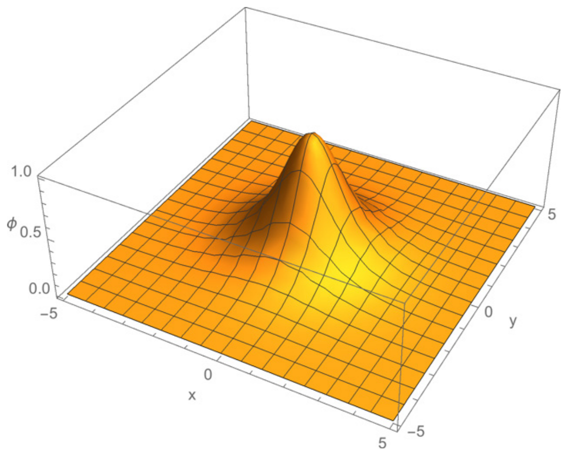


**Figure 3:** Symmetric solution of the potential; the same as it is shown in Fig. 2 presented in Vukcevic & Popovic (2020).

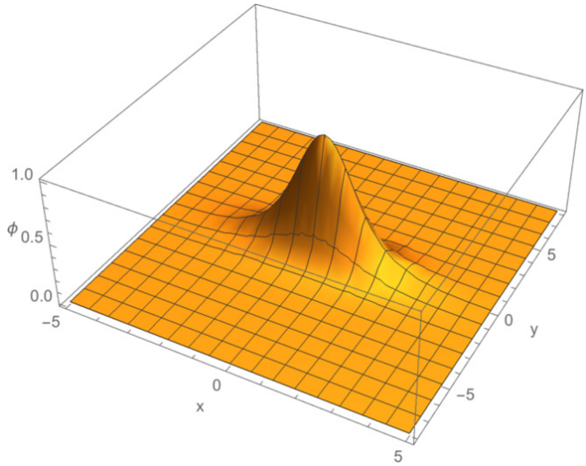


**Figure 4:** Asymmetric solution of the potential close to pole elongated along x axis; the same as it is shown in Fig. 3 presented in Vukcevic & Popovic (2020).

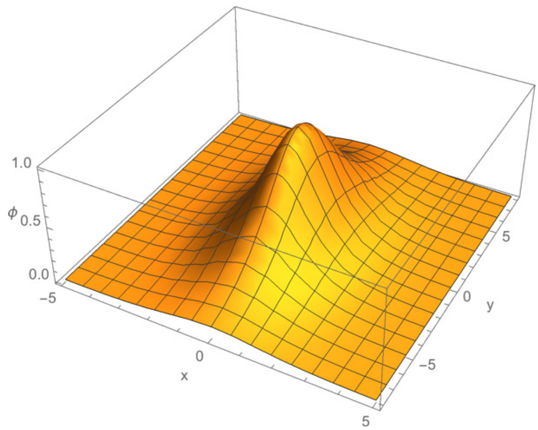


**Figure 5:** Asymmetric solution of the potential in the Equator vicinity elongated along y axis; the same as it is shown in Fig. 4 presented in Vukcevic & Popovic (2020).

## 4. RESULTS

**Ionosphere D layer.** Within this region, located 50– 100 km from the Earth's surface, we can assume that the contribution of charged particles can be neglected, so the ponderomotive force effects are small compared to the Coriolis force effects (Gershman, 1974). This means that it is likely to expect a solitary structure for small latitudes, close to the Equator, elongated along the y coordinate, while for high latitudes and at the pole, the soliton is symmetric, the size of the soliton will depend on the density gradient, and its velocity is normalized by $f = f_R = \Omega$ .

**Ionosphere E layer.** This layer is located 100–150 km from the Earth's surface, and one can expect the creation of a soliton at all latitudes higher than $6^0$, since the ponderomotive force is on the order of the Coriolis one. In this case, soliton velocity is defined by $f = 2(\Omega + H)$ at the pole and for latitudes close to the pole. Since the value H has the opposite sign to Ω, the cancelation of the vortex structure is possible when these two terms are on the same order or it is possible to change the moving direction of the soliton structure. Next, the size of the soliton is defined by R and, consequently, by soliton velocity u, which for this case is defined by $2(\Omega + H)$; one expects the size to increase com- pared with the same case for the D layer. As far as the low-latitude case is concerned, the latitudes close to the Equator and the size and velocity of the soliton are dependent on the value H compared to Ω, and even more, the soliton is not symmetric but rather extended along the y axes, since $f = f(x,y)$.

**Ionosphere F layer.** The ionospheric F layer is for heights 150–400km from the Earth's surface. In this case, for all latitudes soliton structure is mainly defined by the value 0.2H. At the pole, the soliton is symmetric and velocity is defined by $f = 2H$, while close to the Equator one can expect a soliton elongated along the x axis, but moving in the opposite direction compared to the E and D layers, since $f = f(x,y) = f_H$.

## 5. CONCLUSIONS

A series of direct observations of such soliton structures are carried out either from the Earth's surface or onboard the satellites. We have summarized all possible soliton structure formations at different latitudes, as well as at different ionospheric layers. The soliton size and velocity are constant but defined by different values of ionospheric parameters.

We hope that this model will be used in explanations of the ionosphere structures as well as in testing the physics background of complex ionosphere simulations. This model can be used not only to model the ionosphere structure, but also for different astrophysical systems, e.g., accretion disks, where the thickness effects could be very important. Therefore, finite thickness effects should be taken into account. However, this approach can be improved by trying to find out the correlation between soliton structure dynamics and other methods used to identify the ionospheric anomalies. Also, it would be of great importance to investigate the stability of the soliton structure as the subject of small disturbances and apply it to the study of the interaction between the solitons within different ionospheric layers. All of these mentioned issues will be considered in further research.

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